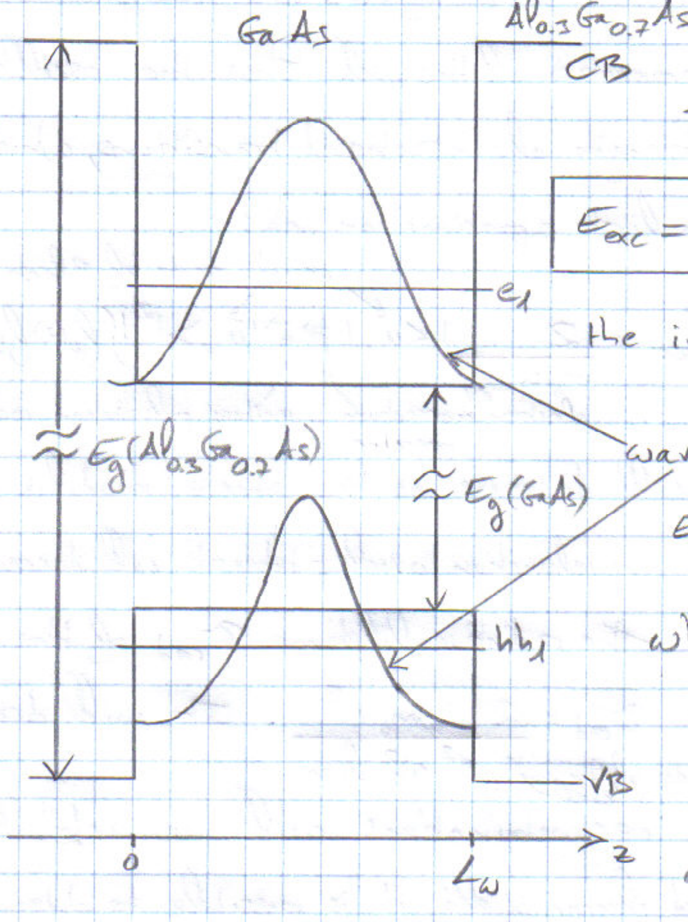
Series _ Electronic and optical semiconductor devices

Series 8 _ Confined levels in a triangular potential well: signature of the quantum confined Stark effect

1 _



It is possible to show that $Ry^{2D} = 4Ry^{3D}$

$$\boxed{E_{exc} = E_g(GaAs) + E_{e_1} + E_{hh_1} - Ry^{2D}}$$

and in the infinite barrier approximation we get:

$$E_{exc} = E_g(GaAs) + \frac{\pi^2\hbar^2}{2m_e^* L_w^2} + \frac{\pi^2\hbar^2}{2m_{hh}^* L_w^2} - Ry^{2D}$$

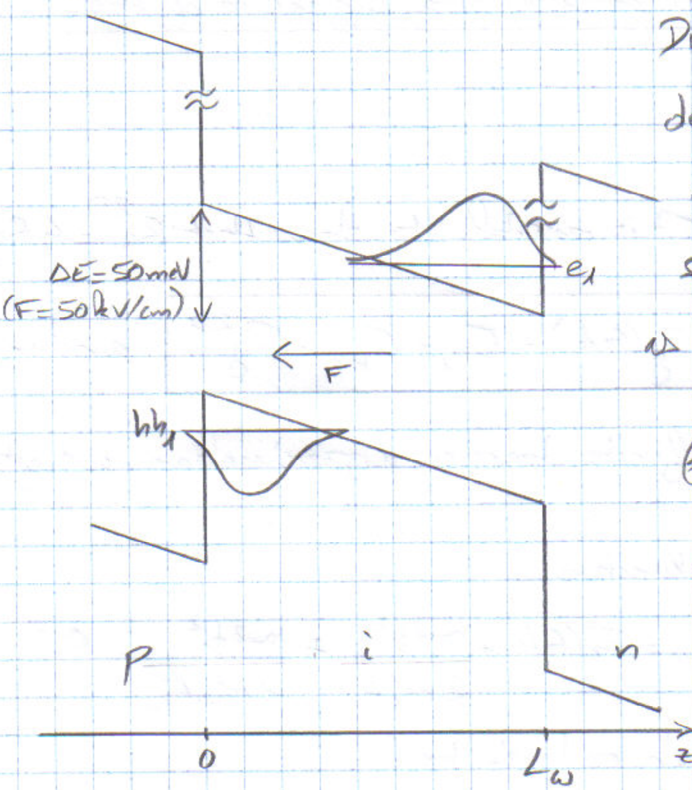
which reduces to:

$$E_{exc} = E_g(GaAs) + \frac{\pi^2\hbar^2}{2\mu_{exc} L_w^2} - Ry^{2D}$$

where $\frac{1}{\mu_{exc}} = \frac{1}{m_e^*} + \frac{1}{m_{hh}^*}$ is the reduced mass of the exciton. Though being more compact, the last expression hides part of the physics. Indeed, from the second equation it is directly seen that the confinement energy of electrons is larger than that of holes due to the lighter mass of the latter in the growth direction. Another consequence of the heavier mass of holes is their stronger localization along z.

2 _ It is seen that the applied electric field induces a spatial separation of electron and hole wavefunctions with a localization occurring at the energetically more favorable extrema of the wells. Consequently holes get localized toward the bottom of the well (substrate side) and the electrons

get localized on the surface side in this configuration (p-i-n diode under forward bias).



Due to the electric field, the wavefunction overlap decreases, which affects the absorption and emission properties of the well. Thus the oscillator strength of interband transitions, which writes as a first approximation as:

$$f_{osc} = \frac{2}{m_0 \hbar \omega_{cv}} |\langle u_c | \vec{p}\,\vec{e} | u_v \rangle|^2 \left|\int f_e(z) f_h(z) dz\right|^2$$

periodic parts of Bloch waves

electron momentum operator

unitary polarization vector

will decrease as the square modulus of the wavefunction overlap integral will decrease.

The radiative lifetime τ_{rad} of this optical transition will also be modified as $\tau_{rad} = \frac{2\pi \varepsilon_0 m_0 c^3}{n_{op} e^2 \omega_{cv}^2 f_{osc}}$. It will drastically increase indicating that radiative recombinations will be inhibited.

optical refractive index of the well

Through a modulation of the applied electric field, it is possible to obtain devices exhibiting a sudden transition from a strongly to a weakly absorbing state. Such devices, which exhibit an electro-optic bistability, are called SEED (for self electro-optic effect device) and can be used as optical memory.

3- The built-in electric field can be deduced from the linear emission energy dependence as a function of well thickness for $L_\omega > 2$ nm.

$L_\omega = 2.6$ nm $\rightarrow$ $E = 3.53$ eV and $L_\omega = 8$ nm $\rightarrow$ $E = 3.155$ eV

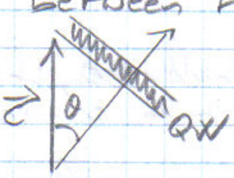
As a result $F = \frac{\Delta E}{\Delta L} = \frac{3.53 - 3.155}{(8-2.6)\times 10^{-7}} \sim 695$ kV/cm

A striking feature of the QCSE lies in the fact that the QW emission energy can be lower than that of the bulk materials (here GaN) for thicknesses > 3 nm. However, the lateral confinement will ensure a non-zero wavefunction overlap even if the latter is very small for thick wells.

$$\boxed{E_{exc} = E_g^{3D} + E_{e_1} + E_{hh_1} - Ry^{2D} - eFL_w}$$

4- Under a high injection level (either under a strong injection current as it is the case for laser diodes or under a high optical power density), electron-hole pairs which are trapped in the well will act against the built-in electric field which will be progressively screened. In III-nitrides the 2D carrier density n_{2D} must be such that $n_{2D} > 2 \times 10^{12}\ cm^{-2}$ to have a noticeable effect. Another possibility consists in doping the barriers. Ionized impurities will provide free carriers to the system which will be trapped in the well and subsequently they will form dipoles with the ionized atoms in the barriers acting against the built-in field.

For structures having the c axis parallel to the plane of the wells, there is no polarization change at the heterostructure interfaces so that no built-in electric field is present (and thus no QCSE). We therefore have QWs with a square potential shape (flat band conditions). For intermediate cases (semi-polar orientations), the field is decreased by a factor $\cos\theta$ where θ is the angle between the c-axis and the growth direction of the well of interest.



$$F_{QW} = F_{\vec{z}} \cos\theta$$

5- The 1D Schrödinger equation within a band subject to an electric field is such that: $-\frac{\hbar^2}{2m^*}\frac{d^2 f_n}{dz^2}(z) + eFz\, f_n(z) = E_n f_n(z)$ (A)

6- $z = \frac{1}{eF}E_n - \left(\frac{\hbar^2}{2m^* eF}\right)\eta$ and $\beta = \frac{2m^*}{\hbar^2}$

equation (A) $\rightarrow$ $-\beta^{-1}\frac{d^2 f_n(z)}{dz^2} + (eFz - E_n)\, f_n(z) = 0$

$$\frac{d^2 f_n}{dz^2} = \frac{d}{dz}\frac{df_n}{dz} = \frac{d}{d\eta}\frac{d\eta}{dz} \times \frac{df_n}{d\eta}\frac{d\eta}{dz} = \frac{d^2 f_n}{d\eta^2}\left(\frac{d\eta}{dz}\right)^2$$

$\frac{dz}{d\eta} = -\beta^{-1/3}(eF)^{-1/3} \Rightarrow \left(\frac{dz}{d\eta}\right)^2 = \beta^{-2/3}(eF)^{-2/3}$ so that we get:

$$-\beta^{-1}\beta^{+2/3}(eF)^{2/3}\frac{d^2 f_n}{d\eta^2} + \left(eF\left(\frac{E_n}{eF} - \beta^{-1/3}(eF)^{-1/3}\eta\right) - E_n\right) f_n = 0$$

which leads to $\boxed{\frac{d^2 f_n(\eta)}{d\eta^2} + \eta f_n(\eta) = 0}$

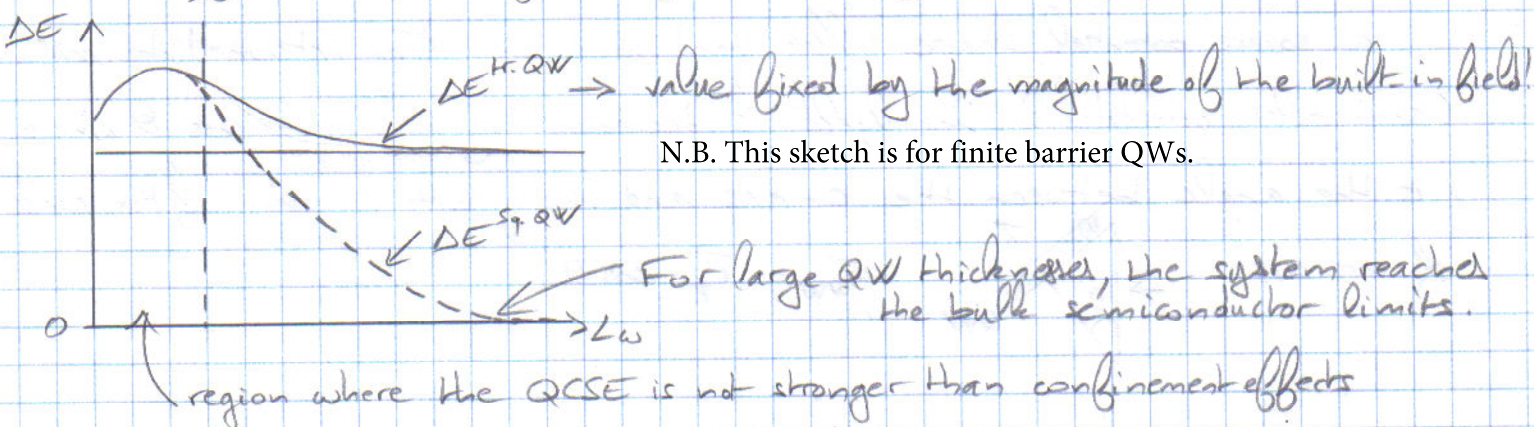
7_ For electrons, the potential is reversed so that the change of variable $z \to -z + L_\omega$ must be performed.

8_ For a square QW within the infinite barrier approximation, the solution of confined energy levels is $E_{n'} = \frac{\hbar^2}{2m^*}\left(\frac{n'\pi}{L_\omega}\right)^2$ with $n' \in \mathbb{N}^*$

Thus $\Delta E_{21}^{Sq.QW} = E_2 - E_1 = \beta^{-1}\left[\left(\frac{2\pi}{L_\omega}\right)^2 - \left(\frac{\pi}{L_\omega}\right)^2\right] = 3\beta^{-1}\left(\frac{\pi}{L_\omega}\right)^2$

For a triangular QW we get $\Delta E_{10}^{tr.QW} = -\beta^{-1/3}(eF)^{2/3}[a_1 - a_0]$

and $\Delta E_{10}^{tr.QW}$ = constant for a given value of F.



N.B. This sketch is for finite barrier QWs.

Transitions $E_2 \to E_1$ and $E_1 \to E_0$ within a given band are called intersubband transitions. Contrary to interband transitions, ISB transitions are independent (as a first approximation) of the nature of the materials forming the well and the barriers. They depend both on L_ω and F. Such a property is used to realize quantum cascade lasers (QCLs) which operate in the infrared ($\lambda \in 3\_100\,\mu m$).

Note that in this last question the same optical transitions are compared since for the square QW $E_{21} \equiv$ ISB transition between the 1st excited level ($n'=2$) and the fundamental level ($n'=1$) and for the triangular QW $E_{10} \equiv$ ISB transition between excited state ($n=1$) and fundamental level ($n=0$).

Further readings

① D.A.B. Miller et al., Phys. Rev. B 32, 1043 (1985)
② N. Grandjean et al., J. Appl. Phys. 86, 3714 (1999).
③ G. Bastard, "Wave mechanics applied to semiconductor heterostructures", Les Editions de Physique, Les Ulis, 1988.